Bulgarian Academy of Sciences. Space Research Institute
Aerospace Research in Bulgaria. 17.2003. Sofia

A VARIATIONAL PROBLEM FOR TIME OPTIMIZATION OF CUMULATIVE CHARGE FOR PSEUDOMETEORITE PARTICLES

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Abstract

A variational problem with unconditional extremum for time maximization of forming shaped charge pseudometeoritical clouds is formulated. The maximal time action of pseudometeoritical clouds particles is used as input parameter. An Euler equation is derived and an intermediate integral for the varied parameter - the profile function of the inside lining is obtained.

The high performance and the small overall dimensions of cumulative charges used to form pseudometeorite particles combined with their low price make them a perspective means to test body robustness of various spacecraft in laboratory conditions. In this connection, a topical problem is studying the options for enhancing the efficiency of cumulative charges by optimizing the impact of the pseudometeorite cloud formed by them. One of the optimized parameters is the time of cloud impact on the craft's protection barrier. Here, two parameters are accounted for: the original deformation with the cumulative lining's collapse as well as the subsequent deformation with the cloud's movement, which enhances the adequacy of the selected physical model.

Using hydrodynamic cumulation theory [1], we shall analyze the metal lining's plane-radial scheme of explosive deformation for the cases where cumulative charge cloud is formed, whose geometry can be described by the following equations in a Cartesian coordinate system yOx [2]: $y=F(x)$;

$y = \Phi(x)$; $y = \varphi(x)$ and $y=f(x)$ for the body, charge and line couple, accordingly. All functions are continuous, smooth, the last two functions have positive first derivatives.

The following expression is known for the stress-load coefficient in a fixed charge section, as a function of charge geometry [1, 2]:

$$\beta = \beta(x) = \frac{o_1 F^2 \Phi 2 + o_2 F^2 \varphi^2 + o_3 \Phi^4 + o_4 \Phi^2 \varphi^2 + o_5 \varphi^4}{o_6 F^2 \varphi^2 + o_7 F^2 f^2 + o_8 \Phi^2 \varphi^2 + o_9 \Phi^2 f^2 + o_{10} \varphi^4 + o_{11} \varphi^2 f^2 + o_{12} f^4};$$

where $o_1, \ldots, o_{12}$ – some ratios of the densities of the body, explosive and lining materials.

Let us consider the original (with collapse) and the subsequent (with the cloud's movement) deformation of the lining - Fig.1. Points A and B launch an inside surface element of the lining $y=f(x)$ with length dx, to a distance x from the origin of the coordinate system (top of lining). The collapse velocities of this element's both ends are $W_0(x)$ and $W_0(x) + dW_0(x)$, accordingly. In time:

$$t_{AC} = \frac{f(x)}{W_0(x)}$$

point A reaches axis $0x$ and the element forms with axis $0x$ collapse angle $\alpha(x)$. The cumulative cloud's compactness and velocity depend on this angle's value [3]. For this time, B point is passes distance:

$$BB' = (t_{AC} - \frac{dx}{D})(W_0(x) + dW_0(x))$$

being apart from axis $0x$ by the distance:

$$B'C' = f + df - BB'$$

and forming angle $\alpha(x)$:

$$\alpha(x) = arctg \frac{B'C'}{dx}$$

where

$$t_{B'C'} = \frac{B'C'}{W_0(x) + dW_0(x)}.$$

With the element's full collapse the cut-off $C'K'$ is formed, with length (to simplify the expression (x) is omitted):

$$C'K' = \frac{W_1}{W_0 + dW_0}\left[f + df - \left(\frac{f}{W_0} - \frac{dx}{D} \right)(W_0 + dW_0) \right] - dx$$

,

where $W_1(x) = W_0/tg(x)$ – the element's launch velocity with collapse. Let us demand that: $tg\alpha = K = const$, where K is the tangent of angle α at which one may purposefully influence the cloud's compactness or discreteness [3, 4].

Then, the initial relative deformation in the end of the collapse will be:

(1) $$\varepsilon_0(x) = \frac{C'K'}{dx}.$$

The cut-off $K''C''$ is determined by expression:

$$K''C''(x,t) = W_1(x)t(x) - W_1(x + dx)\left(t(x) - \frac{dx}{d} \right) - dx$$

But $$t(x) = t - \left(\frac{x}{d} - \frac{f}{W_0} \right).$$

Then, the deformation may be expressed as:

$$\varepsilon(x,t) = \frac{K''C''(x,t) - \sqrt{dx^2 - df^2}}{\sqrt{dx^2 - df^2}}.$$

Finally, after transformations we obtain:

(2) $$\varepsilon(x,t) = \frac{\frac{dW_1}{dx}\left(\frac{x}{D} + \frac{f}{W_0} - t \right) + \frac{W_1}{D} - \frac{f'^2}{2} - 2}{1 + \frac{f'^2}{2}} = [\varepsilon],$$

where $[\varepsilon]$ – the stress-load deformation's marginal value for the lining material.

Then, the general time functional of cumulative lining collapse, accounting for the allowed lining material deformation will have the form:

(3) $$T = \int_0^H S(x, f, f')dx$$

where

$$S(x,f,f')=\frac{1}{D}+\frac{2}{D}\frac{f'\beta(2+\beta)-f\beta'}{\beta\sqrt{\beta(2+\beta)}}+\frac{1}{D}\frac{\beta'^2-\beta''\beta'(2+\beta)+\beta'^2(1-\beta)}{\beta'^2}-$$

$$-\frac{(2+\beta)}{\beta'^2\sqrt{\beta(2+\beta)}}tg\alpha\frac{1}{D}\{(1+[\varepsilon])[\beta(2+\beta)(f''\beta'-f'\beta'')+(1-\beta)f'\beta'^2]-$$

$$-2(2+[\varepsilon])[\beta''\beta(2+\beta)-\beta'^2(1-\beta)]\}.$$

With the adopted symbols, Euler's equation will be [5]:

(4) $$S'_f-S''_{xf'}-S''_{ff'}\frac{df}{dx}-S''_{f'f'}\frac{d^2f}{dx^2}=0;$$

(5) $f(0)=f0; f(H)=fH,$

where H is the cumulative lining's altitude.

Since in the right-hand side of (3) x does not participate immediately, in the left-hand side of (4) the second term, $S''_{xf'}=0$, is missing, and we can write the intermediate integral [5]:

(6) $$S-S'_{f'}\frac{df}{dx}=C_1,$$

which yields:

(7)

$$\frac{1}{D}+\frac{2}{D}\frac{f'\beta(2+\beta)-f\beta'}{\beta\sqrt{\beta(2+\beta)}}+\frac{1}{D}\frac{\beta'^2-\beta''\beta'(2+\beta)+\beta'^2(1-\beta)}{\beta'^2}-$$

$$-\frac{1}{D}tg\alpha(1+[\varepsilon])(2+\beta)\frac{\beta(2+\beta)(f''\beta'-f'\beta'')+(1-\beta)f'\beta'^2}{\beta'^2\sqrt{\beta(2+\beta)}}-f'\left\{\frac{2}{D}\frac{\beta(2+\beta)-f(\beta')'_{f'}}{\beta\sqrt{\beta(2+\beta)}}+\right.$$

$$+\frac{1}{D}\frac{1}{\beta'^4}\left\{\left\{2\beta(\beta')'_{f'}-(2+\beta)\left[(\beta'')'_{f'}\beta'+\beta''(\beta')'_{f'}\right]+2(1-\beta)\beta'(\beta')'_{f'}\right\}\beta'^2-2[\beta'^2-\right.$$

$$-\beta''\beta(2+\beta)+\beta'^2(1-\beta)\big]\beta'(\beta')'_{f'}\Big\}-\frac{1}{D}tg\frac{\alpha}{2}(1+[\varepsilon])\frac{2+\beta}{\sqrt{\beta(2+\beta)}}\frac{1}{\beta^4}\{\{\beta(2+\beta)\Big[f''(\beta')'_{f'}-$$

$$-\left(\beta''+f'(\beta'')'_{f'}\right)\Big]+(1-\beta)\left(\beta'^2+2f'\beta'(\beta')'_{f'}\right)\Big\}\beta'^2-2[\beta(2+\beta)(f''\beta'-f'\beta'')+$$

$$-(1-\beta)f'\beta'^2\big]\beta'(\beta')'_{f'}+\frac{2}{D}tg\frac{\alpha}{2}(2+[\varepsilon])\frac{2+\beta}{\sqrt{\beta(2+\beta)}}\frac{1}{\beta'^4}\Big\{\Big[\beta\Big(2+\beta(\beta'')'_{f'}-$$

$$-2(1-\beta)\beta'(\beta')'_{f'}\Big]\beta'^2-2\big[\beta''\beta(2+\beta)-\beta'^2(1-\beta)\big]\beta'(\beta')'_{f'}\Big\}=C_1$$

where C_1 is the integration constant;

β', β'', $(\beta')'_{f'}$, $(\beta'')'_{f'}$ are derivatives of the stress-load coefficient.

As a result, the variational problem for time optimization of a pseudometeorite cloud cumulative charge effect is formulated and an intermediate integral (7) is obtained. The problem requires locating the unconditional maximum of functional (3) with edge conditions (5). The varied parameter is the function describing the inside surface lining profile. It makes no problem to change this varied parameter for another parameter of the cumulative charge geometry. The problem is interesting in that, in functional (3), there is an option to control the cloud's digitization rate by varying parameter $[\varepsilon]$.

References

1. Physics of Detonation. K.P. Stanjukovitch, Moscow, Science, 1973, 703.
2. H. I. Hristov, Ph.D. Thesis, Tula University, Tula, Russia, 1993, 146.
3. N. A. Zlatin, About Maximum Speeds of a Solid Condensed Cumulative Charge Jets. Problem of Mathematics and Mechanics. Novosibirsk, Science, 1983.
4. S. K. Godunov, A. A. Derybas, I. V. Maly, About Influencing Material Viscosity in the Process of Jets Formation at Impact of Metallic Laminas. - Physics of Combustion and Detonation. Novosibirsk, Science, 1975, No.1, 3-18.
5. J. B. Zeldovitch, A. D. Mishkis. Members of Applied Mathematics. Moscow, Science, 1967.

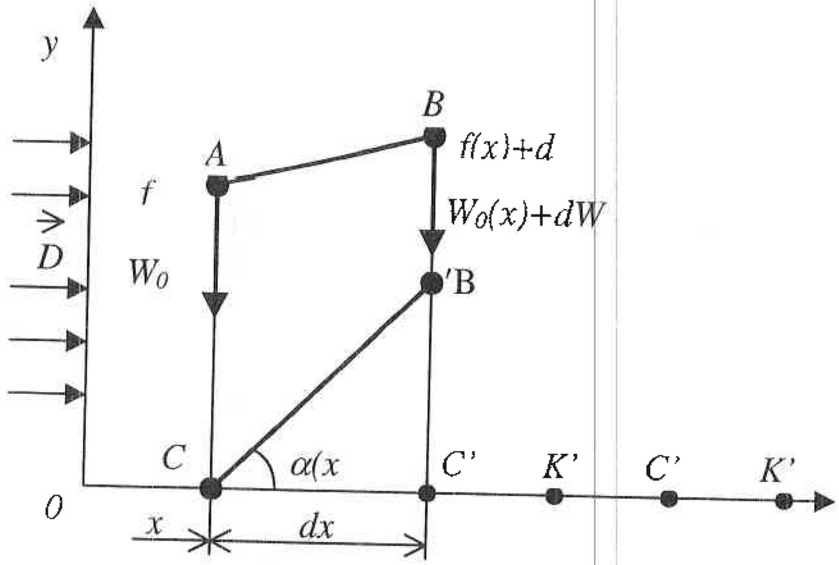

Fig. 1. The scheme of the initial and subsequent deformation of the cumulative lining.

ЗАДАЧА ЗА ОПТИМИЗАЦИЯ ПО ВРЕМЕТО НА ЕФЕКТА ОТ ДЕЙСТВИЕТО НА ПСЕВДОМЕТЕОРИТЕН ОБЛАК

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Резюме

Формулирана е вариационна задача с безусловен екстремум за оптимизация на кумулативен заряд за псевдометеоритни частици по параметър максимално време на действие на облака от псевдометеоритни частици, изведено е уравнение на Ойлер и е получен промеждутъчен интеграл за параметъра на вариране – функцията на профила на вътрешната повърхност на облицовката.